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Measurement-Based Radar Waveform Optimization Using the Ambiguity Function and Spectral Mask

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***Abstract*—This paper demonstrates the optimization of a radar waveform based on ambiguity function measurements and spectral mask criteria. Compliance with spectral mask criteria is becoming ever-critical in radar systems due to the increased number of devices using the wireless spectrum. The capability of reducing ambiguity at certain range-Doppler combinations is useful in enhancing the detection capability of radar systems. This paper shows the combined optimization for both ambiguity function and spectral mask criteria based on time-domain measurements taken with a high-speed oscilloscope. A benefit of optimizing based on measurements is that the transmitter power amplifier's output waveform can be used, allowing the results of nonlinear distortion to be factored into the optimization process.**

I. Introduction

Spectrum constraints on radar systems continue to grow more stringent. As mid-course is nearing in the *National Broadband Plan* of the United States, requiring 500 MHz of spectrum to be re-allocated to wireless broadband by 2020, radar systems are required to perform their all-important detection functions with more limited bandwidth. The interference ramifications of not complying with spectral mask criteria continue to grow more severe as more devices are squeezed into the available spectrum. The spectral mask is the defined range of power versus frequency within which the output signal must remain [1].

Waveform optimization for radar systems has been studied in multiple works. Variable-modulus techniques [2] and constant-modulus techniques such as continuous-phase modulation [3-4] and piecewise linear chirp optimization [5] are among the approaches found in the literature. Chirps provide useful benefit because they can easily be compressed, allowing the benefit of enhanced range resolution [6]. Linear frequency-modulated chirps can be tuned by perturbing the phase of the terms of the Fourier Series [7].

Our approach is to minimize the maximum of the ambiguity function's value over multiple specified points in the range-Doppler plane. Holtzmann and Thorp demonstrate the use of the ambiguity function as a weighted error criterion for optimization [8], and Wong and Chung have applied genetic algorithms for ambiguity function minimization in specific range-Doppler combinations [9]. Sussman has applied least-squares optimization to find the best radar waveform efficiently [10]. Regarding the spectral spreading of waveforms, Blunt *et al.* have demonstrated the use of continuous-phase modulation to reduce spectral spreading [3-4].

Our work is unique in that it examines the optimization of chirp waveform bandwidth to (1) minimize the ambiguity function's magnitude over specified range-Doppler combinations and (2) maintain spectral mask compliance. Our initial work has demonstrated the validity of this approach using simulations [11], and a recent follow-on paper has shown the ability to calculate the ambiguity function from measured time-domain oscilloscope data [12]. The present paper uses the oscilloscope-measured data to perform, based on time-domain measurement data, the *minimax* optimization demonstrated in simulation in [11].

II. The Radar Ambiguity Function

The radar ambiguity function is given by the following expression [6]:

$$\chi(\tau, u) = \int_{t=-\infty}^{\infty} x(t)x^*(t-\tau)e^{-j2\pi ut}dt, \qquad (1)$$

where $x(t)$ is the transmitted signal, τ is the difference in time from the actual time delay associated with the target, and u is the frequency difference in Hertz from the actual Doppler frequency shift based on the target's true velocity. The ambiguity function serves practically as a measure of the output of the correlator at deviations in range and Doppler from the true target. In radar detection, the waveform is usually designed to place nonzero ambiguities at range-Doppler combinations that will not pass erroneous information to the target. For example, if a secondary target exists at a range-Doppler combination of high ambiguity, the reflected wave could pass through the correlator and indicate a target is present at the (0,0) range-Doppler combination. Such a situation could cause issues in target detection and could complicate air traffic control and/or battlefield scenarios, for example. Thus, it is normally desired to minimize the ambiguity at range-Doppler combinations corresponding to the expected locations of other targets or interferers.

Figure 1 shows an example of the simulated ambiguity function magnitude for a linear FM chirp [11]. As predicted by Skolnik, the tilt of the ambiguity ridge in the range-Doppler plane can be changed by altering the ratio of bandwidth to time-width of the pulse.

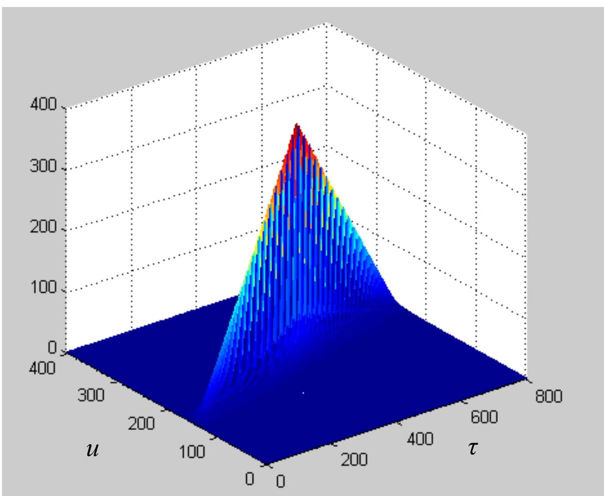


Fig. 1. Ambiguity function magnitude for a linear FM chirp [11]

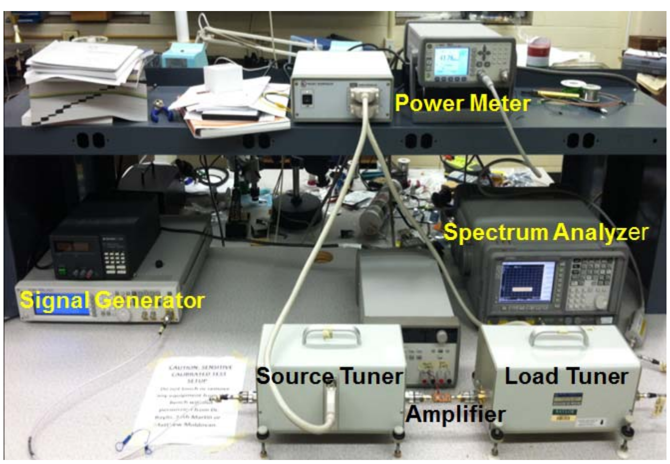


Fig. 2. Measurement test bench

III. Measurement Test Setup

The measurement setup for bench-top measurements used in this paper includes an Agilent N5182A signal generator, an Agilent E4407B spectrum analyzer, and a LeCroy Wavemaster 8500 oscilloscope. The signal generator is used to convert the programmed chirps to the carrier frequency of 3.3 GHz for the experiments, and the oscilloscope collects the time-domain data for calculation of the ambiguity function. Figure 2 shows the nonlinear measurement test setup in the Baylor research laboratory.

As detailed in [12], the waveform is measured with the oscilloscope in the time domain, and then converted to baseband for the optimization. This downconversion allows the number of samples to be reduced for analyzing the signal. For baseband analysis, the signal is separated into its in-phase (I) and quadrature (Q) components.

IV. Search for the Optimum Chirp

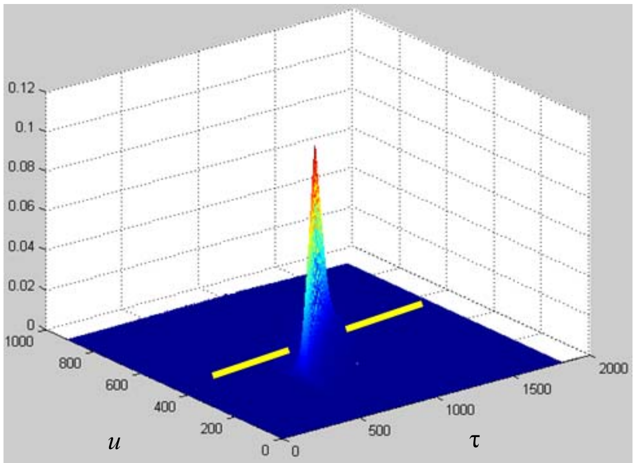
The search was performed on the test bench to find the best linear FM chirp taken from a catalogue of candidates. The waveform catalogue consisted of linear FM chirps with bandwidths varying from 0 to 5 MHz in steps of 250 kHz. The time width of all chirps was held constant at 10 microseconds. For this simple example, this means that 21 bandwidth options are available. In addition, both up- and down-chirps were considered, providing 42 different chirps from which to choose the optimum waveform (in general, up- and down-chirps have related, but distinctly different ambiguity functions).

In a method that parallels the simulation demonstration of [11], each chirp is first measured for spectral mask compliance with the spectrum analyzer. If the chirp does not meet the spectral mask criteria, it is removed from consideration. From the chirps that meet the spectral mask criteria, the chirp providing the smallest measured maximum ambiguity function value over specified range-Doppler combinations of interest is selected as the optimum chirp. As mentioned, the selected range-Doppler combinations for minimization are usually those for which a misinterpretation would be particularly undesirable or detrimental. This is a type of *minimax* optimization, and was chosen for this optimization problem due to the fact that a particularly large ambiguity at any point could be detrimental to the success of the detection, even if the average ambiguity over the range-Doppler combinations of interest is low.

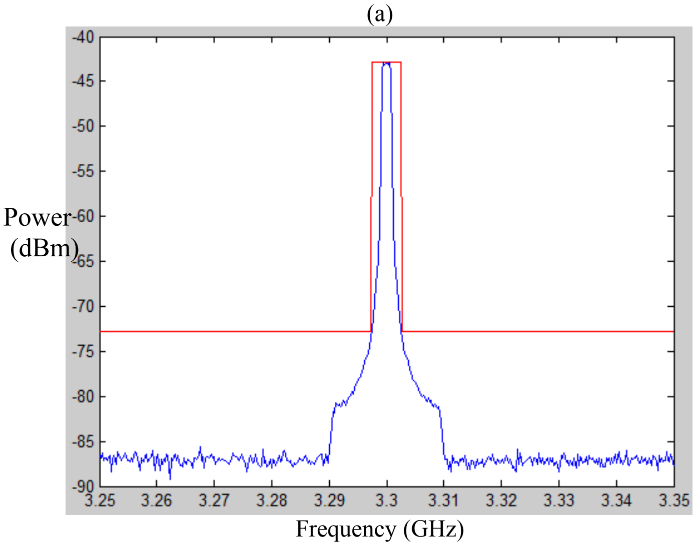
V. Measurement Optimization Results

The optimization was based on the ambiguity function, as calculated from time-domain oscilloscope measurements, and the spectrum as measured by the spectrum analyzer.

For the first test ("Optimization 1") example, the points for ambiguity minimization were selected along the τ axis. This serves the practical case of a range-oriented radar, a radar that has excellent range resolution, but is likely to possess ambiguities along the Doppler axis. Figure 3 shows the measurement results for this ambiguity function. The line in Fig. 3(a) showing the points for minimization in the minimax optimization is spread along the τ axis, indicating that a range radar is desired. The ambiguity ridge is nearly head-on in the view shown in Fig. 3(a). While ideally the ambiguity of a range radar would be aligned on the Doppler axis, such a situation would violate spectral masks. It can be seen that the result of this optimization, as expected, is that the widest bandwidth possible while meeting spectral mask requirements (Fig. 3(b)) is selected as the optimum. This example aligns very well with intuitive expectations.



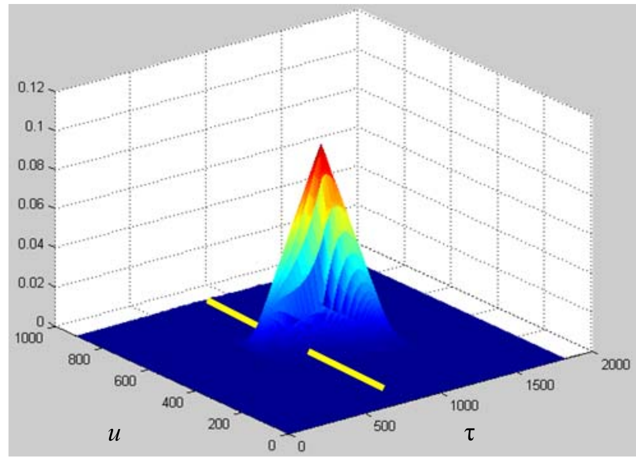
(a)



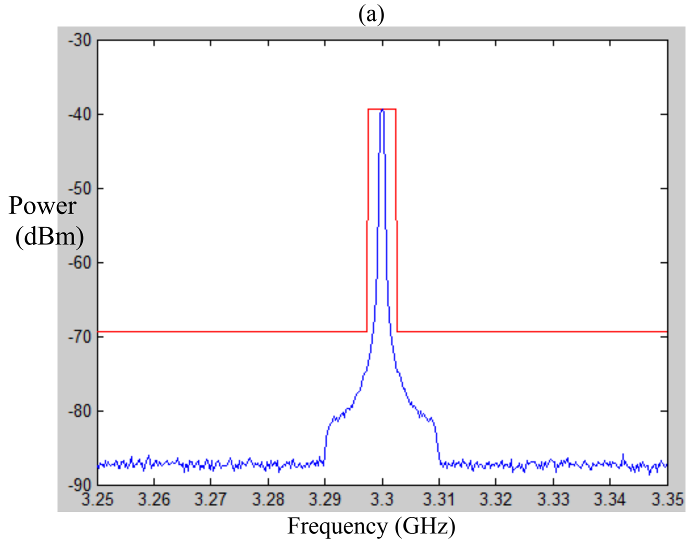
(b)

Fig. 3. (a) Ambiguity function magnitude with lines denoting minimization points for the minimax optimization, and (b) baseband spectrum and spectral mask for the best chirp in Optimization 1. The chosen chirp here is a 1.75 MHz bandwidth down chirp.

For the second test (“Optimization 2”), the results are shown in Fig. 4. The minimization points for this example were chosen along a constant value of u. This result is more focused on Doppler resolution than range resolution because low bandwidth chirps are the only spectrally compliant chirps with low ambiguity at the minimization points. The ridge in the ambiguity function splits the two minimization lines in Fig. 4(a). Fig. 4(b) shows that this chirp is well inside the spectral mask, which corresponds with expectations for this case. While our spectral mask was arbitrarily selected for



(a)

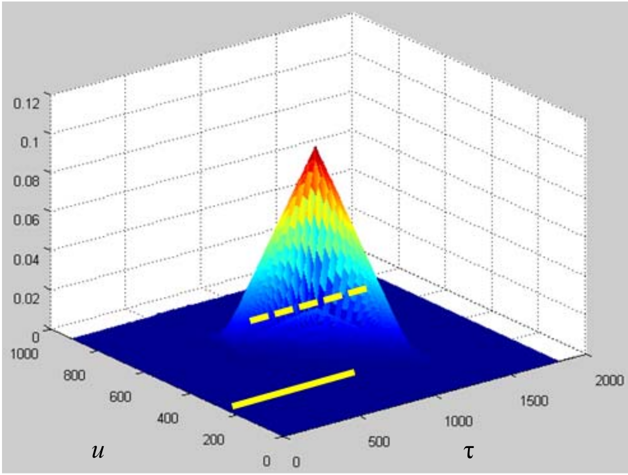


(b)

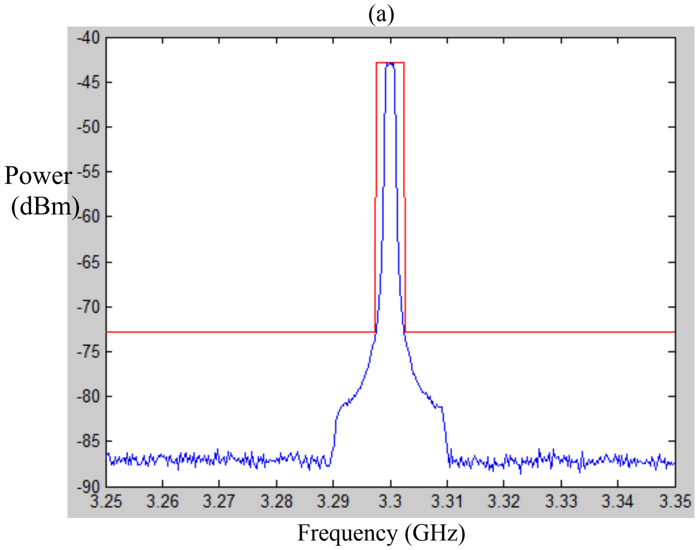
Fig. 4. (a) Ambiguity function magnitude with lines denoting minimization points for the minimax optimization, and (b) baseband spectrum and spectral mask for the best chirp in Optimization 2. The chosen chirp here is a 0.75 MHz bandwidth up chirp.

experimental purposes, spectral masks for many radar systems in the United States are set forth by the National Telecommunications and Information Administration [13].

For the third test (“Optimization 3”), the results are shown in Figure 5. The minimization points for this example were chosen in the front and back corners of the ambiguity function, indicating that the desired result is an up chirp focused on range resolution. Like in optimization 1, the chosen chirp has the maximum bandwidth which passes the spectral mask. The ridge in ambiguity shown in Fig. 5(a) is stretched from the left corner toward the right corner of the graph, once again avoiding the minimization lines. Again, this result fits intuitive expectations.



(a)



(b)

Fig. 5. (a) Ambiguity function magnitude with lines denoting minimization points for the minimax optimization, and (b) baseband spectrum and spectral mask for the best chirp in Optimization 3. The chosen chirp here is a 1.75 MHz bandwidth up chirp.

VI. Conclusions

The significant contribution of this paper is to demonstrate in a measurement-based optimization approach the usefulness of minimax optimization to select a waveform providing the most useful ambiguity function, given selected range-Doppler combinations for which it is desired to minimize the ambiguity. The results shown in this paper lay the groundwork for future innovations in the area of waveform optimization based on output waveforms from the radar transmitter power amplifier, which may incur significant distortion, resulting in spectral spreading and, in some possible cases, modification of the ambiguity function.

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References

[1] J. de Graaf, H. Faust, J. Alatishe, and S. Talapatra, "Generation of Spectrally Confined Transmitted Radar Waveforms," Proceedings of the IEEE Conference on Radar, 2006, pp. 76-83.

[2] R. Chen and B. Cantrell, "Highly Bandlimited Radar Signals," Proceedings of the 2002 IEEE Radar Conference, pp. 220-226.

[3] S. Blunt, M. Cook, E. Perrins, and J. de Graaf, "CPM-Based Radar Waveforms for Efficiently Bandlimiting a Transmitted Spectrum," 2009 IEEE Radar Conference, pp. 1-6.

[4] M. Cook, M.S. Thesis: *CPM-Based Radar Waveforms for Efficiently Bandlimiting a Transmitted Spectrum*, University of Kansas, April 2010.

[5] M. Moldovan, C. Baylis, R.J. Marks II, L. Cohen, and J. de Graaf, "Radar Chirp Waveform Selection and Circuit Optimization Using ACPR Load-Pull Measurements," IEEE Wireless and Microwave Technology Conference, Cocoa Beach, Florida, April 2012.

[6] M. Skolnik, *Introduction to Radar Systems*, Third Edition, McGraw-Hill, 2001.

[7] L. Patton, *On the Satisfaction of Modulus and Ambiguity Function Constraints in Radar Waveform Optimization for Detection*, Doctoral Disseration, Wright State University, 2007.

[8] J. Holtzman and J. Thorp, "Optimum Signals for Radar," *IEEE Transactions on Aerospace and Electronic Systems,* Vol. 5, No. 6, pp. 898-905, November 1969.

[9] K. Wong and W.-K. Chung, "Pulse-Diverse Radar/Sonar FSK-PSK Waveform Design to Emphasize/De-Emphasize Designated Doppler-Delay Sectors," IEEE Radar Conference, Alexandria, Virginia, May 200, pp. 745-749.

[10] S. Sussman, "Least-Square Synthesis of Radar Ambiguity Functions," *IRE Transactions on Information Theory*, Vol. 8, No. 3, pp. 246-254, April 1962.

[11] M. Fellows, C. Baylis, L. Cohen, and R.J. Marks II, "Radar Waveform Optimization to Minimize Spectral Spreading and Achieve Target Detection," 2013 Texas Symposium on Wireless and Microwave Circuits and Systems, Waco, Texas, April 2013.

[12] M. Fellows, C. Baylis, L. Cohen, and R.J. Marks II, "Calculation of the Radar Ambiguity Function from Time-Domain Measurement Data for Real-Time, Amplifier-in-the-Loop Waveform Optimization," Automatic RF Techniques Group Conference, Columbus, Ohio, December 2013.

[13] "Radar Spectrum Engineering Criteria," U.S. National Telecommunications and Information Administration, January 2008.